

BDF Tutorials: Network based stress testing

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Avoiding coordinated overreaction



Outline

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- ▶ Systemic Risk
- ▶ Solvency contagion, liquidity contagion, and credit deterioration
- ▶ Mechanisms of solvency contagion
 - ▶ Ex-post clearing and partial exogenous recovery
 - ▶ Ex-post clearing and full endogenous recovery
 - ▶ Ex-post clearing and partial endogenous recovery
- ▶ Mechanisms of distress contagion
 - ▶ Ex-ante local uncertainty and no recovery
 - ▶ Ex-ante local uncertainty and endogenous recovery
- ▶ Exercises on financial contagion



Systemic risk

Systemic Risk. The risk of a loss involving a large fraction of the financial system and comparable with its total volume.



Financial contagion

Channels of financial contagion that cause systemic risk:

- ▶ **Solvency contagion.** The risk of being insolvent caused by a counterparty or by a market loss.
- ▶ **Distress contagion.** The risk of credit deterioration caused by a counterparty or by a market loss.
- ▶ **Liquidity contagion.** The risk of a lack of liquidity caused by a counterparty or by a market loss.



A general framework for financial contagion

We define a simple financial system composed by a set \mathcal{B} of financial institutions with balance sheets consisting of:

- ▶ external assets, $\mathbf{A}^{(e)}$,
- ▶ external liabilities, $\mathbf{L}^{(e)}$,
- ▶ interbank liabilities. $(L_{ij})_{ij}^{|\mathcal{B}|}$.

Namely $\mathcal{F} = \{\mathcal{B}, \mathbf{A}^{(e)}, \mathbf{L}^{(e)}, (L_{ij})_{ij}^{|\mathcal{B}|}\}$.

Due to financial contagion, the net value of a financial institution, identified with a vector of equities \mathbf{E} , depends on the network of financial relations between the institutions.

In particular, for many relevant cases we can write the following:

$$E_i = A_i^{(e)} V^{(e)}(E_i) - L_i^{(e)} + \sum_j A_{ij} V(E_j) - \sum_j L_{ij} \quad (1)$$



Mechanisms of solvency contagion

Furfine

- ▶ It accounts for **ex-post** clearing and **partial exogenous recovery**,
- ▶ An insolvent bank is defined by the condition $E_i < 0$
- ▶ An insolvent bank repays a fraction R of its liabilities.

$$E_i = A_i^{(e)} - L_i^{(e)} + \sum_j A_{ij} (\mathbb{1}_{E_j > 0} + R\mathbb{1}_{E_j \leq 0}) - \sum_j L_{ij} \quad (2)$$



Mechanisms of solvency contagion

Eisenberg and Noe

- ▶ It accounts for **ex-post** clearing and **full endogenous recovery**,
- ▶ finds p_i , i.e. the payment vector resulting from the clearing procedure,
- ▶ defines \bar{p}_i *total interbank liabilities*,
- ▶ *relative liabilities matrix*: i 's liability to j as proportion of i 's total
 $\Pi_{ij} = L_{ij}/\bar{p}_i$ if $\bar{p}_i > 0$ and $\Pi_{ij} = 0$ otherwise

$$p_i = \min \left[\bar{p}_i, \sum_j \Pi_{ji} p_j + A_i^{(e)} - L_i^{(e)} \right] \quad (3)$$



Mechanisms of solvency contagion

Rogers and Veraart

- ▶ It accounts for **ex-post** clearing and **partial endogenous recovery**,
- ▶ includes, effectively, fire-sales of both external and interbank assets of insolvent banks,
- ▶ finds p_i , i.e. the payment vector resulting from the clearing procedure,
- ▶ defines \bar{p}_i *total interbank liabilities*,
- ▶ *relative liabilities matrix*: i 's liability to j as proportion of i 's total $\Pi_{ij} = L_{ij}/\bar{p}_i$ if $\bar{p}_i > 0$ and $\Pi_{ij} = 0$ otherwise

$$p_i = \min \left[\bar{p}_i, \beta(p_i) \sum_j \Pi_{ji} p_j + \alpha(p_i) A_i^e - L_i^{(e)} \right] \quad (4)$$



Mechanisms of distress contagion

DebtRank

- ▶ It accounts for **ex-ante** credit deterioration and effectively considers **no recovery**,
- ▶ considers the uncertainty due to solvency risk in the future,
- ▶ defines the leverage matrix Λ ,
- ▶ finds h_i , i.e. the vulnerability vector \mathbf{h} , resulting from the DebtRank algorithm.

$$h_i(t+1) = \min \left[1, h_i(t) + \sum_j \Lambda_{ij}(t) [h_j(t) - h_j(t-1)] \right] \quad (5)$$



Mechanisms of distress contagion

Endogenous DebtRank

- ▶ It accounts for **ex-ante** credit deterioration and considers **partial endogenous recovery**,
- ▶ considers the uncertainty due to solvency risk in the future,
- ▶ defines the stochastic process followed by a bank's counterparties external assets,
- ▶ finds a vector of equities which is consistent with a local Merton pricing of the interbank assets,
- ▶ converges to the corresponding ex-post model when time to maturity goes to zero.

$$E_i = A_i^{(e)} - L_i^{(e)} + \sum_j A_{ij} \mathbb{E} \left[\left(\mathbb{1}_{E_j^T > 0} + \left(1 + \frac{E_j^T}{\bar{p}_j} \right) \mathbb{1}_{E_j^T \leq 0} \right) \middle| E_j \right] - \sum_j L_{ij}$$



Summary

Mechanisms of financial contagion

- ▶ Furfine, $V(E) = \mathbb{1}_{E_j > 0} + R\mathbb{1}_{E_j \leq 0}$,
- ▶ Eisenberg-Noe, $V(E) = \mathbb{1}_{E > 0} + (1 + E/\bar{p})\mathbb{1}_{E \leq 0}$,
- ▶ Rogers-Veraart,
 $V(E_i, E_j) = (\mathbb{1}_{E_i > 0} + \beta\mathbb{1}_{E_i \leq 0}) [\mathbb{1}_{E_j > 0} + (1 + E/\bar{p})\mathbb{1}_{E_j \leq 0}]$,
- ▶ DebtRank, $V(E) = \frac{E}{E(0)}\mathbb{1}_{E > 0} + R\mathbb{1}_{E \leq 0}$,
- ▶ Endogenous DebtRank,
 $V(E) = \mathbb{E} \left[\left(\mathbb{1}_{E^T > 0} + \left(1 + \frac{E^T}{\bar{p}_j} \right) \mathbb{1}_{E^T \leq 0} \right) \middle| E \right]$,

